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Time Evolution of the Electric Field Using the Rapid Expansion Method with Pseudo-Spectral Evaluation of Spatial Derivat

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Summary

The 3D three-component time response of the electric wave field can be simulated numerically by considering the spatial and time responses separately and employing numerical methods best suited for each. Here we use the rapid expansion method to develop the 3-component electric wave field time response from the spatial responses found using a pseudo-spectral method. The results are free of numerical dispersion and accurate to the Nyquist frequency in time and space. The method is intrinsically parallel which leads to computational efficiency. The method developed here is suitable for modeling transient EM data acquired on land or at sea. Numerical results compare favorably with the analytic response and 1D methods even though the computations are intrinsically 3D.

Introduction

Flexible 3D three-component full-bandwidth modeling is important in controlled-source electromagnetic (CSEM) exploration for survey design and for inversion of the data. Unlike the seismic method, there is no ray theory for EM, and interpretation of the data is totally dependent on the accuracy of the modeling. Mittet (2010) provides an excellent overview of finite-difference time domain approaches to the problem of modeling EM data and analyzes the trade-off between speed and accuracy before proposing his own improved finite-difference approach. All finite-difference schemes have accuracy limited by the approximations of the spatial and temporal derivatives.

We solve for the 3D three-component E-field directly using an explicit time evolution called the rapid expansion method (REM), proposed by Tal-Ezer (1986, 1989), combined with a pseudo-spectral evaluation for the spatial derivatives (Fornberg, 1987; Fornberg, 1988). Our method is based on an approach proposed by Carcione (2006), who solved the 2D problem for a magnetic source. Pestana and Stoffa (2010) have applied REM to the time evolution of the 3D acoustic wave equation. Unlike finite difference methods, our method is accurate to the Nyquist frequency in time and the Nyquist wavenumber in space. Further, the method is unconditionally stable. We incorporate the analytic solution for an impulsive electric dipole source in a full space as our initial condition to begin the time evolution of the 3D three-component E-field. The REM expands the exponential of the spatial operator using Chebyshev polynomials, which possess a wave-like character that shows where the energy is propagating. After the integration of the polynomials with modified Bessel function weights, the true arrival time and diffuse character of the energy become obvious.

We present the derivation of the algorithm, closely following the notation of Carcione (2006). Then we compute the result for a dipole buried in a whole space and compare it with the analytical solution to demonstrate the accuracy of the method. We then apply the scheme to a 3D model to demonstrate its potential and the wave-like character of the Chebyshev polynomials.

Theory

We follow the development and notation of Carcione (2006), but extend Carcione's two-dimensional (x, z) analysis for dipole magnetic sources to three dimensions (x, y, z) for an electric dipole source. The electric field vector $\mathbf{E}(x, y, z, t)$ satisfies the diffusion equation

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{1}{\mu\sigma} \nabla \times \nabla \times \mathbf{E} - \sigma \frac{\partial \mathbf{J}}{\partial t}, \quad (1)$$

in which $\mu = \mu_0 = 4\pi 10^{-7}$ H/m is the magnetic permeability, σ (S/m) is the electrical conductivity, and \mathbf{J} is the current source. Carcione (2006) recognized that equation (1) is of the form

$$\frac{\partial \mathbf{w}}{\partial t} = \mathbf{G}\mathbf{w} + \mathbf{s}, \quad (2)$$

where \mathbf{w} is the wavefield (the electric or magnetic field), \mathbf{s} is the source term, and the 3D operator

$$\mathbf{G} = -\frac{1}{\mu\sigma} \nabla \times \nabla = -\frac{1}{\mu\sigma} \begin{pmatrix} -(\partial_y^2 + \partial_z^2) & \partial_y \partial_x & \partial_z \partial_x \\ \partial_x \partial_y & -(\partial_x^2 + \partial_z^2) & \partial_z \partial_y \\ \partial_x \partial_z & \partial_y \partial_z & -(\partial_y^2 + \partial_x^2) \end{pmatrix}. \quad (3)$$

After discretization, the solution of equation (3) satisfying the initial condition $\mathbf{w}(0) = \mathbf{w}_0$ is formally given by (Carcione 2006, eq. 18):

$$\mathbf{w}_N(t) = \exp(t\mathbf{G}_N)\mathbf{w}_N^0 + \int_0^t \exp(\tau\mathbf{G}_N)\mathbf{s}_N(t-\tau)d\tau. \quad (4)$$

Using the Chebyshev expansion of $\exp(x)$, in the absence of sources, the discrete solution (Carcione 2006) is

$$\mathbf{w}_N^M(t) = \sum_{k=0}^M b_k(t)T_k(\mathbf{F}_N)\mathbf{w}_N^0, \text{ where} \quad (5)$$

$$\mathbf{F}_N = \frac{1}{a}(\mathbf{G}_N + a\mathbf{I}), \quad (6)$$

\mathbf{I} is the identity matrix of dimension 3, and a is the absolute value of the eigenvalue of matrix \mathbf{G}_N having the largest negative real part (See Carcione, 2006). Here

$$b_k(t) = c_k \exp(-at)I_k(at), \quad (7)$$

$c_0 = 1$, $c_k = 2$, for $k \geq 1$, and I_k is the modified Bessel function. The value of $T_k(\mathbf{F}_N)\mathbf{w}_N^0$ is computed using the recurrence relation for Chebyshev polynomials,

$$T_k(u) = 2uT_{k-1}(u) - T_{k-2}(u), \quad k \geq 2, \text{ and} \quad (8)$$

$$T_0(u) = 1, \quad T_1(u) = u. \quad (9)$$

(Abramowitz and Stegun, 1972, and Carcione, 2006). The maximum wavenumber components are the Nyquist wavenumbers, which for grid spacings Δx , Δy and Δz are $k_x = \pi/\Delta x$, $k_y = \pi/\Delta y$ and $k_z = \pi/\Delta z$. They are related to the highest harmonic of the spatial Fourier transform. Hence, the value of a is

$$a = \frac{\pi^2}{\mu\sigma_{\min}} \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2} \right), \quad (10)$$

where σ_{\min} is the lowest value of conductivity in the model. a has units of s^{-1} . As Tal-Ezer (1989, eq. 4.13) shows, the polynomial order should be $O(\sqrt{at})$. Carcione (2006) found that

$$M = \beta\sqrt{at} \quad (11)$$

is enough to obtain stability and accuracy, with β in the range 5 to 6. As an initial field we use the time derivative of the analytic expression for the step response of an x -directed dipole at the origin of a full-space, given by Ward and Hohmann (1987). The impulse response initial field is then

$$\mathbf{w}_0 = \frac{M\sigma^{1/2}\mu^{3/2}}{8\pi^{3/2}t^{5/2}} \exp\left(-\frac{\mu\sigma r^2}{4t}\right) \left[\left(1 - \frac{\mu\sigma}{4t}(y^2 + z^2)\right)\mathbf{u}_x + \frac{\mu\sigma xy}{4t}\mathbf{u}_y + \frac{\mu\sigma xz}{4t}\mathbf{u}_z \right], \quad (12)$$

with M the dipole moment, and \mathbf{u}_x , \mathbf{u}_y and \mathbf{u}_z unit vectors in the x , y and z directions and $r = (x^2 + y^2 + z^2)^{1/2}$.

Examples

Figure 1 shows the three components of the electric field computed with REM at two positions in the computational homogeneous isotropic cube. Comparison with the analytic response demonstrates the accuracy of the computation.

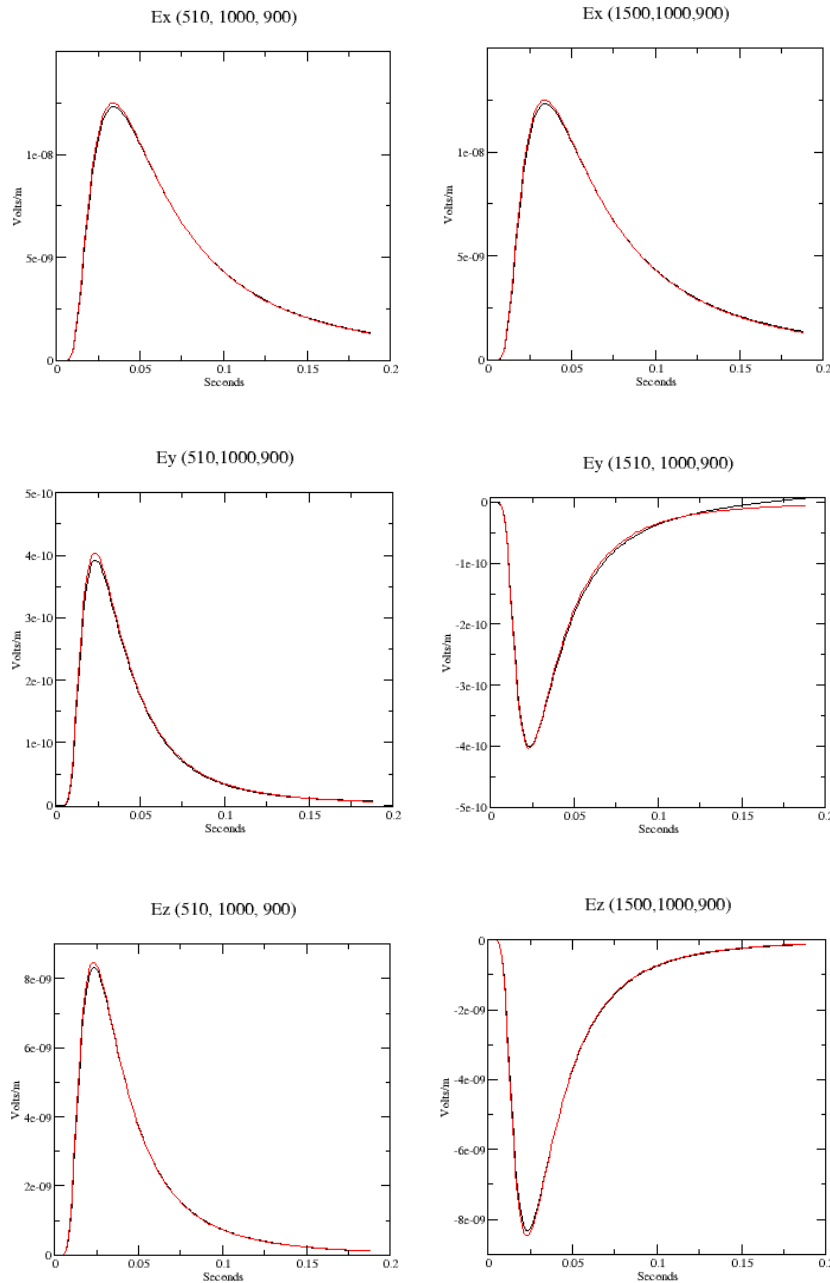


Figure 1 Comparison of pseudo-spectral (black) and analytic (red) responses for E_x , E_y , and E_z components at (510, 1000, 900) on the left and (1500, 1000, 900) on the right. The origin of coordinates is at one corner of the cube. The cube has $200 \times 200 \times 200$ grid points; $\Delta x = \Delta y = \Delta z = 10\text{m}$; the x -directed dipole source is at (1005, 1005, 1005).

Figure 2 shows four Chebyshev polynomials, illustrating their wave-like character in a 3D model consisting of three layers: 0-2200 m - $0.33333 \Omega\text{m}$; 2200-2280 m - $1 \Omega\text{m}$; 2280-4000 - $2 \Omega\text{m}$. A 50 m up-thrown vertical fault was added, centred at $x=1850\text{m}$, $y=2000\text{m}$, with 300m extent in x and 200 m in y . A 100 m x -directed dipole source at (2005, 2005, 2005) was used to start the propagation.

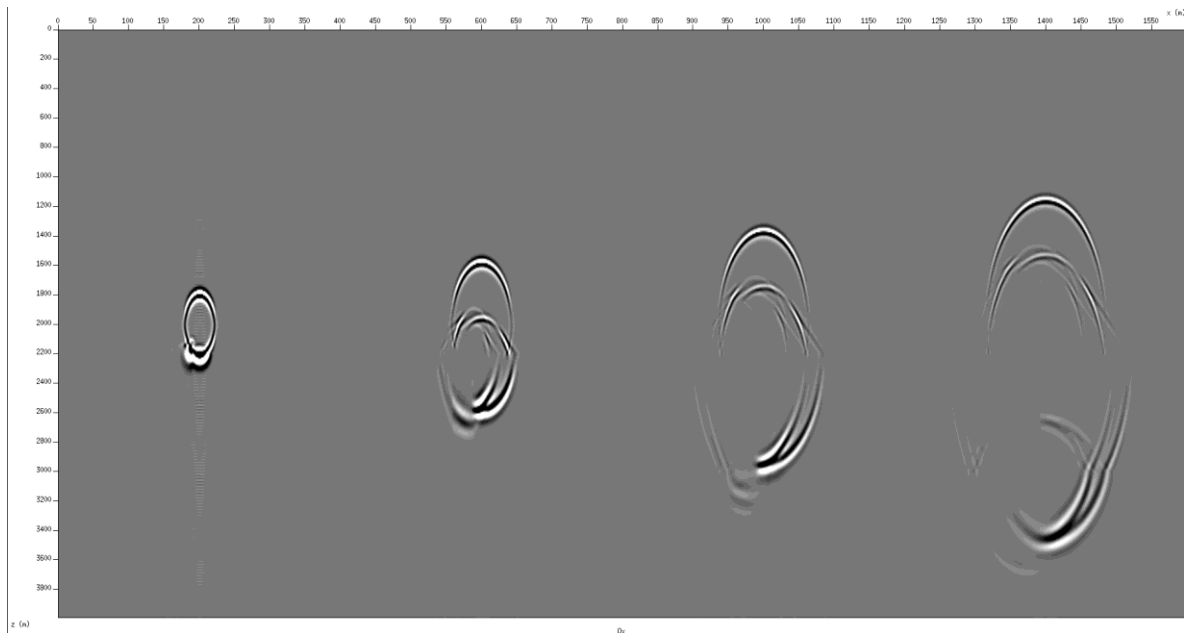


Figure 2 2D slice at $y = 2000$ m through Chebyshev polynomials of E_x for $k = 200, 400, 600$ and 800 in a 3D 400 by 400 by 400 model computation with 10 m sampling in all directions.

Conclusions

We have demonstrated that the new flexible 3D three-component full-bandwidth modelling method for transient CSEM data is accurate by comparing it with the analytic response to a point dipole in a full space. Similar accuracy is obtained in comparisons with 1D modelling (not shown here). The diffusive field is a weighted sum of Chebyshev polynomials, which exhibit a wave-like character and are sensitive to small perturbations in the medium. The method we developed can be implemented in parallel at two levels. First, each component of the electric field can be computed independently and their updated E field components exchanged only once per polynomial evaluation. Second, within each of these global processes, local parallelism is achieved over all computational loops. This implementation combined with parallel FFTs makes the method computationally feasible.

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